On Deployment Requirements of Overlaid LTE Transceivers

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***Abstract*—It has been shown through simulation results [14]–[16] that the interference adaptive dynamic channel allocation (IA-DCA) scheme is a promising resource allocation strategy in time/frequency-division multiple-access (TDMA/FDMA) communication systems. The major obstacle in analyzing IA-DCA is the computation of cochannel interference without the constraint of conventional channel reuse factors. To overcome this difficulty, one needs a computationally efficient representation which can approximate the interference distribution accurately. For this purpose, a concept called channel reuse zone (CRZ) is introduced. Based on this new concept, both downlink and uplink cochannnel interference are computed with two different propagation models, namely, a simplified deterministic model and a shadowing model. The results are then used to calculate the outage probability of the idealized, interference adaptive maximum packing (IAMP) scheme. Finally, as a significant contribution, an asymptotic performance bound for the two-way IA-DCA strategy is derived.**

***Index Terms*—Channel assignment, cochannel interference, resource allocation, wireless networking.**

I. Introduction

ONE OF the most difficult problems in the wireless communications area is the scarcity of radio resources. In order to meet the challenge of the rapidly increasing demand for the system capacity, researchers are exploring new resource-saving techniques in all layers. Channel allocation is a critical task performed by the network layer in bandwidth-limited networks, i.e., time/frequency-division multiple-access (TDMA/FDMA) systems.

In most existing cellular networks, a fixed number of channels is assigned to each cell. The same channel cannot be reused within a reuse distance in order to guarantee the carrier-to-interference ratio (CIR) of a connection between a base station (BS)/mobile station (MS) pair. This is called the fixed channel assignment (FCA). Various dynamic channel allocation (DCA) strategies, which aim to enhance the efficiency of bandwidth usage, have been devised since the idea was first suggested in early 1970's [1], [2]. In the basic DCA, any channel can be assigned to any cell as long as the CIR constraint is satisfied. Furthermore, other network dynamics can be exploited.

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The existing DCA strategies are classified into three categories based on the type of network dynamics they exploit and the representation of the CIR constraint they employ. **Traffic adaptive DCA (TA-DCA)** strategies [3]–[7] take advantage of the unevenness of traffic volume among cells and use concepts from the first-generation FDMA networks, such as "channel reuse factor" and "compatibility matrix," to represent the CIR constraint. Another type of DCA which exploits the mobility of users is location adaptive DCA (LA-DCA) strategy [8], [9]. In the typical implementation of LA-DCA, i.e., *reuse partitioning*, a cell is split into a number of concentric subcells and the CIR constraint is approximated by different channel reuse factors for different subcell groups. The third type of DCA which assigns channels based on real-time interference measurement is called **interference adaptive DCA (IA-DCA)** strategies [10]–[13]. There is no formal representation of the CIR constraint in IA-DCA schemes. More comprehensive overviews on DCA can be found in [17]–[19].

Since the received CIR is the ultimate constraint in the resource allocation problem, it is reasonable to predict that IA-DCA methods will achieve more capacity gain than other channel allocation techniques. In fact, many simulation studies have confirmed this prediction [13]–[16]. However, theoretical analysis of IA-DCA has not been thoroughly addressed in the literature. The idea of approaching the upper bound of the system capacity by dividing a cell into infinite number of concentric zones has appeared in the literature [20]–[22]. However, since the method of calculating the perimeter of an arbitrary zone for a channel reuse factor other than the classical values (1, 3, 4, 7, ...) was unknown, the authors in these references could only assume hexagonal or circular zone shape and could not alleviate the limitation of the conventional channel reuse factor. This means that the bounds they obtained were for LA-DCA rather than IA-DCA schemes. The performance bound of IA-DCA in one-dimensional (1-D) highway networks was investigated in [23] (without power control) and [24] (with power control). However, it is hard to extend this decent work to the two-dimensional (2-D) planar case.

The main obstacle for the performance analysis of IA-DCA in planar networks is the representation of cochannel interference (CCI) constraint without approximation. The existing literature on CCI computation addresses the code-division multiple-access (CDMA) systems [25]–[27] and FCA-based

TDMA/FDMA: system [20], [27]. In this paper, we present a method for computing the concentric zone boundaries corresponding to any nontrivial value of channel reuse factor.[1] The result is: not only are we able to get rid of the confine of the conventional concept of the channel reuse factor, but also we can extend our investigation from the downlink-only DCA, deterministic propagation model (without shadowing) to the case of both the link and shadow fading models.

The contribution of this paper can be summarized as follows. First, a novel concept called the *channel reuse zone* (CRZ) and its extension to the shadowing model, the *extended CRZ* (ECRZ), are introduced. Furthermore, a CRZ/ECRZ structure called *the concentric* CRZ *structure* is described. This structure represents the most compact channel reuse pattern under the CIR constraint. Second, an idealized IA-DCA scheme called *interference adaptive maximum packing* (IAMP) algorithm is defined. The motivation of the study on IAMP is to find the performance bound of the IA-DCA strategies. The CCI and outage probability are calculated for IAMP in both downlink and uplink channels. Finally, a performance measure called the *asymtotic probability of assignment failure*, introduced in [20], is used as a benchmark and its lower bound (the upper performance bound) for IA-DCA strategies is derived by applying results obtained in the first two parts. All of the computations and derivations consider both the deterministic and shadowing models.

The paper is organized as follows. Notation, assumptions and propagation models are presented in Section II. Section III focuses on the deterministic model. First, the CRZ concept and the IAMP algorithm are given, and then the computation of CCI is presented. The results for the deterministic case are extended for the shadowing model in Section IV. Section V deals with the derivation of the lower bound of the asymtotic probability of assignment failure for IT-DCA. strategies. Numerical results and interpretations are presented in Section VI.

II. Notation, Models, and Assumptions

The typical cellular network architecture is considered in this paper. A *cell* is a geographic area corresponding to the radio-service area of a BS.[2] An MS communicates with an assigned BS through a *channel* with an assigned transmit power level. A channel might be a frequency band in FDMA systems or a time slot in TDMA systems. The effect of power control is not considered in this paper.

The CIR constraint rules that a channel p can be assigned to a BS/MS link only if $(C/I)_p \leq \gamma$, where $(C/I)_p$ is the CIR of channel p and γ is the desired CIR threshold. A downlink (uplink)-only DCA algorithm assigns a channel pair if the CIR on the **downlink (uplink)** channel is above the γ. This algorithm assumes that the CIR's on both channels are identical. A **two-way DCA** scheme assigns a pair of channels only if *both* downlink and uplink channels meet the CIR constraint.

A. Notation

The following notation is used in the paper.

R	Radius of a cell.
A_c	Area of a cell.
D	Distance between two neighboring BS's. $D = \sqrt{3}R$.
r_{Ji}	Distance between BS J and MS i.
$P^d(P^u)$	Received downlink (uplink) local mean signal power.
L_{ji}	Shadowing effect of the received signal.
α	Path-loss exponent.
U_j	Number of ongoing calls in cell j.
F	Average rate of blocked and dropped calls.
M	Total number of channels in the system.
V	Number of cells in the system.
P_{out}	Outage probability.

B. Propagation Model

The downlink signal power received at MS i from BS J is expressed as

$$P_{Ji}^d = P_0 L_{Ji} r_{ji}^{-\alpha} \tag{1}$$

where P_0 is the received signal power at unit distance. Similarly, the uplink signal power received at BS I from MS j is given as

$$P_{jI}^u = P_0 L_{jI} r_{jI}^{-\alpha}. \tag{2}$$

In our study, we assume that: 1) the system is interference limited, such that back noise power is negligible; 2) the shadow fading L_{Ji}'s are independent lognormal random variables (RV) with a mean of 0 dB and a standard deviation of σ dB[3]; 3) omnidirectional antennas are used; and 4) interchannel interference is negligible, so only cochannel interference is considered.

C. Traffic Model

We assume that the traffic in the system has uniform distribution, hence U_j is a random variable with statistics independent of cell index j. Furthermore, offered traffic in each cell is assumed to be an independent stochastic process with an average arrival rate λ (calls/cell/ s). The mean duration of each call is $1/\mu$ (s). The average offered traffic in a cell is

$$\xi = \lambda/\mu(\text{calls/cell}). \tag{3}$$

We can define the relative traffic load as $p = \xi/M$ [20]. The *probability of assignment failure* is defined as $\nu = E[F]/\xi V$. Therefore, at any snapshot in time, U_J's are independent RV with a mean of $\lambda(1-\nu)/\mu$.

A keen reader may notice that the mobility of mobiles is not explicitly considered in this model. In most cases, the mobility issue makes a huge difference in the design and evaluation of a practical DCA algorithm. Mobility increases the signaling load in the system. It may also lead the DCA designer to give priority to handoff calls at the cost of reducing the system capacity. Nevertheless, mobility is not explicitly modeled in this study due to the following reasons. First, the impact of mobility

[1]A "nontrivial value" means a real number in the range of $[1,\ h_K]$, where h_K is determined by the CIR threshold and the propagation model (see definition in Section III).

[2]All the theoretical results in this paper are independent of the cell shape. For the sake of simplicity, the numerical results in deterministic case (in Section VI) are acquired by assuming hexagonal cells.

[3]The i.i.d. assumption is made for the sake of simplicity. The result can be easily modified for the correlated case by applying formulas in [28].

on system capacity is represented by increased rate of channel seizure attempts (λ) accompanied by shorter channel holding times $(1/\mu)$. The resulting snapshot traffic load ξ in (3) is similar to ξ without mobility. Second, in order to find the performance bound for IA-DCA strategies, one needs an allocation algorithm which is more powerful than any other IA-DCA schemes. For this purpose, we design the idealized IAMP method, which is assumed to have unlimited processing power to perform all of the necessary channel allocations/reallocations instantaneously. Therefore, the probability of handoff failure due to processing delay is zero. Third, the probability of assignment failure ν, the benchmark performance measure in our study, includes both probability of new call blocking and forced termination and does not distinguish between them.

To calculate the performance measures for IA-DCA schemes, one needs to know the maximum number of users the system can accommodate. For this purpose: 1) a representation of the CIR constraint has to be defined in an IA-DCA environment and 2) the outage probability at every point in the cell has to be calculated for the given DCA algorithm and the given representation of the CIR constraint. The calculation of P_{out} depends on the CCI computation. The next two sections are organized as follows: 1) the CIR constraint; 2) the representation of the constraint and definition of the allocation algorithm (IAMP); and 3) the computation of interference and P_{out} for both uplink and downlink.

III. Cochannel Interference Analysis for Deterministic Case

In the deterministic case, shadow fading in (l) and (2) is set to be 0 dB. The downlink and uplink CIR constraints for the cell of interest (denoted by cell 0) are thereby defined as

$$(C/I)^d = \frac{r_{00}^{-\alpha}}{\sum_{J \in \mathcal{I}^d} r_{J0}^{-\alpha}} \geq \gamma \tag{4}$$

and

$$(C/I)^u = \frac{r_{00}^{-\alpha}}{\sum_{j \in \mathcal{I}^u} r_{j0}^{-\alpha}} \geq \gamma \tag{5}$$

respectively, where

and $\mathcal{I}^d = \{$cells sharing the downlink channel with cell $0\}$

$\mathcal{I}^u = \{$cells sharing the uplink channel with cell $0\}$.

A. Representation of the CIR Constraint—Channel Reuse Zones

Imagine the scenario that a mobile 0 on channel moves from BS 0 to the border of the cell in an arbitrary direction. At the beginning, the MS is very close to BS 0 so that (4) holds even if all other cells are transmitting on channel p. When MS 0 moves away from BS 0, its CIR decreases. At a certain point, one of the neighboring cells of cell 0 has to be forbidden from transmitting on channel p, in order to satisfy (4). On its way to the cell border, MS 0 causes more and more cells to be prohibited from using channel p. Since the BS's are discretely located, CRZ's are naturally formed.

Definition 1: **A downlink CRZ** *(CRZ) is a region in a cell where* (4) *holds for the same interference distribution despite of the mobile's location. An* **uplink CRZ** *is a region in a cell where* (5) *holds for the same interference distribution despite of the mobile's location. A* **two-way CRZ** *is a region in which both* (4) *and* (5) *are simultaneously satisfied.*

It is clear that the shape and area of a CRZ are determined by α, γ, and the distribution of cochannel interferers. The number of possible zone partitionings is virtually infinite. A special partitioning of CRZ's is defined below. It turns out, by CCI computation (see Section III-B) that this specific CRZ partitioning consists of a set of concentric zones. Therefore, we call it as a **concentric CRZ structure**. The word zone in the next paragraph stands for either a downlink, uplink, or two-way CRZ.

In the concentric CRZ structure, *zone-1* is defined as the largest CRZ in which a channel can be reused in all the cells without violating the CIR constraint. Then, the cells in the system are divided into $V/2$ groups. Each group has two neighboring cells. *Zone-2* is defined as the *largest* CRZ in which a channel can be reused in one of the two cells in a cell group. The word "largest" means that the cochannel cells from different groups should be distributed as sparsely as possible. By dividing the system into $V/3$ identical groups, with three neighboring cells in each group, we can define two CRZ'S: *Zone-(3/2)* is the largest CRZ in which a channel can be reused in two out of three cells in a cell group. *Zone-3* is the largest CRZ in which a channel can be reused in one out of three cells in a cell group. In general, *Zone*-(n, l) $(n, l$ integers, $n > l > 0)$ is obtained by splitting the system into V/n identical cell groups, each with n cells, and assigning a channel to l cells in each group in such a way that the resulting CRZ covers the largest possible region (or the cochannel cells are distributed most sparsely). Let K be the number of CRZ's in each cell, the Kth zone is denoted as *zone*-h_k, $k = 1, 2, \cdots, K$, where $h_1 = 1$, $h_k > h_{k-1}$.

The physical meaning of h_k is that a channel can be reused in one out of h_k cells within zone-h_k. Therefore, it can be viewed as an extension of the "channel reuse factor." However, it should be emphasized that h_k might take improper fraction values. This implies that K may go to infinity, although the value of h_k is upper bounded by α and γ. Note that for group indexes $n \geq 2$, the layout of the cell group is not unique. For example, four cells may be arranged in a row or stacked in two rows. Even the shape of a cell group which gives the largest CRZ area may change for the same k. However, for a given h_k, the *area* of the *largest* CRZ is identical in every cell since the interference distributions are the same for every cell group. Since *only the area* of CRZ'S, which corresponds to the offered number of calls, is useful in our analysis, the definition of zone-h_k is reasonable.

Figs. 1 and 2 illustrate the relationship between zones in the concentric CRZ structure and the corresponding channel reuse patterns. Numbers in cells represent the distance between the cell of interest and another BS (unit: D). Fig. 1 shows the most sparse cochannel user distribution for a channel used in every

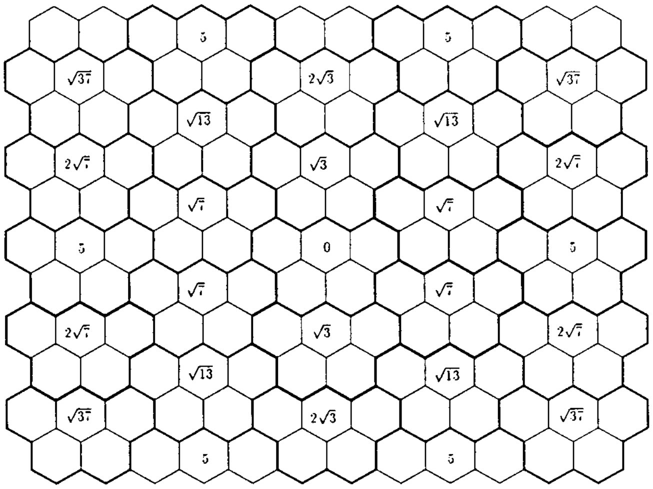

Fig. 1. Channel reuse pattern of concentric CRZ zone-5.

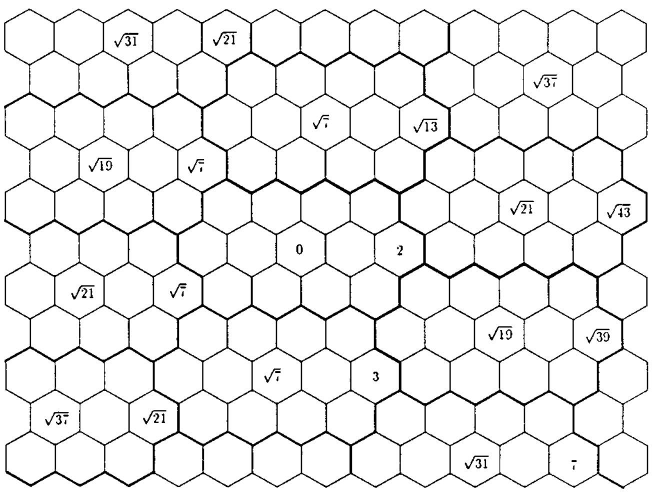

Fig. 2. Channel reuse pattern of concentric CRZ zone-13/2.

one out of five cells. This reuse pattern is utilized to compute the area of CRZ zone-5. Fig. 2 shows the most sparse channel reuse pattern corresponding to zone-13/2.

Definition 2: A **zone**-h_k *user is defined as a mobile located within zone-*h_k *and out of zone-*h_{k-1}. *A* **zone**-h_k **channel** *is a channel assigned to a zone-*h_k *user, which might be either an uplink or a downlink channel.*

The theorem below is the basis for the study on the performance bound of IA-DCA.

Theroem 1: Assume that: 1) traffic is uniformly distributed in each cell and 2) the allocation policy is fair for all cells, i.e., the blocking probability in each cell is the same for equal traffic loads. As the number of CRZ's K *goes to infinity, the concentric CRZ structure represents the CIR constraint of the most compact channel reuse pattern in TDMA/FDMA systems.*

Proof: The most compact channel usage in TDMA/FDMA systems is the one where a channel is reused in all V cells (reuse pattern 1). To use channels in an optimum way, reuse pattern 1 should contain as many users as possible. In other words, reuse pattern 1 should cover the averagely largest possible region in a cell (by assumption a), and the area of this largest region should be identical in every cell (by assumption b). This argument results in the definition of the concentric CRZ zone-1. For the users outside zone-1, the next most compact reuse pattern is when one channel is reused in $(n-1)V/n$ cells, where n is an arbitrarily large positive integer. By the same argument as before, we obtain the definition of zone-$n/(n-1)$. By continuing the same procedure on users outside zone-$n/(n-1)$, one obtains the whole concentric CRZ structure in a straightforward way.

It is apparent that the concentric CRZ structure with finite K is still an approximation to the CIR constraint. However, this structure can approximate the original constraint with an unlimited accuracy by increasing K. The real CIR constraint of the most compact channel reuse pattern will be reached when $K \rightarrow \infty$.

B. The Interference Adaptive Maximum Packing Method

In order to study the performance bound for IA-DCA schemes, every channel is expected to be packed as densely as the CIR constraint allows. Due to the limited processing capability of a real network, no practical DCA scheme is able to take full advantage of the channel reuse pattern represented by the concentric CRZ structure. Therefore, the idea of the well-known *maximum packing* (MP) approach (see [5]) is employed for the purpose of theoretical study. The basic idea of MP is of assuming that there exists a central controller who knows all the system-wide information and has an unlimited computational power. The controller will do all of the necessary channel reallocations in the whole system in order to accommodate a new call. Since MP is originally used in TA-DCA, the method proposed here is called *the IAMP method* to avoid confusion. The following algorithm may be viewed as either downlink/uplink-only or two-way IA-DCA, depending on what type of CRZ zones is used.

- Step 1) Set $k = 1$.
- Step 2) Pick one zone-h_k user from every group of h_k cells, do all the necessary channel reallocation, and pack them into a single channel (or a channel pair in two-way case). The cells occupying the same channel should be in the same location within their own groups (see Figs. 1 and 2). In the two-way case, uplink and downlink channels are assigned such that cells using the same uplink channels are assigned the same downlink channels and vice versa. Continue this process until all zone-h_k users are packed or all available channels are used up.
- Step 3) Increase k by one; go back to step 2 if $k \leq K$, otherwise, stop the algorithm.

The idea of ordering the channel packing is to increase efficiency. Since a zone-h_k channel can only be reused in V/h_k cells, the maximum number of users a channel can support decreases as k increases. Therefore, it is more efficient if channels are assigned to zone-h_k users first, then zone-h_2 users, zone-h_3 users, and so on.

The significance of binding uplink and downlink channels together can be conceived through the following inequality:

$$\begin{aligned} P_{\mathrm{out}}^{nb} &= 1 - \left(1 - P_{\mathrm{out}}^{u}\right)\left(1 - P_{\mathrm{out}}^{d}\right) \\ &\geq \max\left[P_{\mathrm{out}}^{u}, P_{\mathrm{out}}^{d}\right] = P_{\mathrm{out}}^{bd} \end{aligned}$$

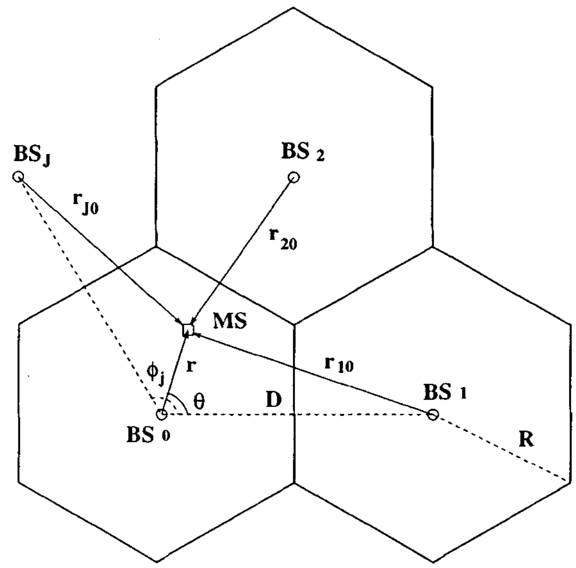

Fig. 3. Computation of downlink cochannel interference.

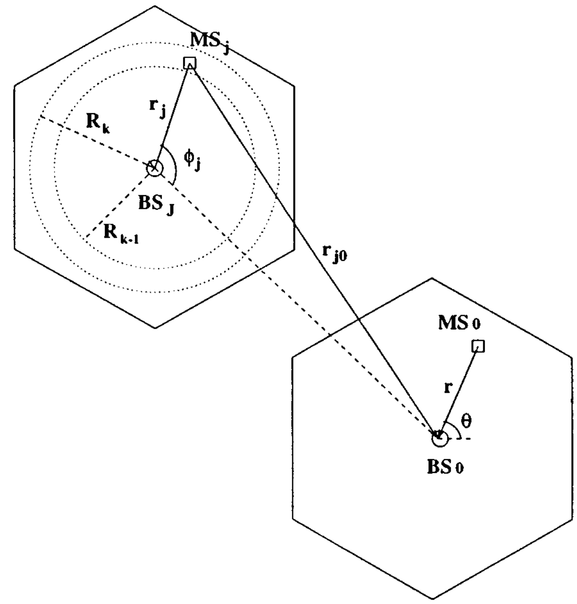

Fig. 4. Computation of uplink cochannel interference.

where P_{out}^{u}, P_{out}^{d}, P_{out}^{bd}, and P_{out}^{ub} are outage probabilities of the uplink channel, downlink channel, two-way channels assigned with binding, and two-way channels assigned without binding, respectively.

Theorem 2: Under the assumptions in Theorem 1, the IAMP method applied to the concentric CRZ structure becomes optimum (in terms of channel use efficiency) in DCA strategies as K goes to infinity.

Proof: From Theorem 1, when K goes to infinity, the concentric CRZ structure represents the most compact channel reuse pattern. In other words, it gives the largest average set of available channels under the CIR constraint. The assumption of unlimited processing power of IAMP implies that if there is an available channel in the system, the IAMP method is able to assign it within an arbitrarily short time period. As the result, the combination of these two factors gives the optimal channel utilization.

C. Computation of Cochannel Interference: Deterministic Model

1) Computation of CIR for General Case: The computation of downlink cochannel interference is explained through Fig. 3. To simplify the notation, let us define $r = r_{00}$ and θ and ϕ_j as shown in Fig. 3. The downlink local mean interference is expressed as

$$\begin{aligned}\overline{P}_{j0}^{d} &= P_0 r_{J0}^{-\alpha}\\ &= P_0\left[w_j^2D^2 + r^2 - 2w_jDr\cos(\phi_j-\theta)\right]^{-(\alpha/2)} \quad (6)\end{aligned}$$

where $w_j = 1, \sqrt{3}, \sqrt{7}, 3, \cdots$, depending on the distance between BS J and BS 0. The downlink CIR at MS 0 is written as

$$(\text{CIR})^{d} = \frac{r^{-\alpha}}{\sum\limits_{j\in\mathcal{I}^d}\left[w_j^2D^2 + r^2 - 2w_jDr\cos(\phi_j-\theta)\right]^{-(\alpha/2)}}. \quad (7)$$

The computation of the uplink interference is illustrated in Fig. 4. Now, the difficulty is that every r_{j0} is an independent RV. Therefore, an equation like (6) which relates $\overline{P}_{j0}^{u}$ with the position of MS 0 does not exist. The main difference between the uplink interference computation in CDMA (see, e.g., [25] and [26]) and this study is that in CDMA the interference from another cell is due to an accumulated effect of average ξ users, whereas in TDMA/FDMA there is up to one interferer in each cell. Therefore, instead of computing the expected *total* interference from a cell, we compute the expected interference power received at BS 0 from MS j as

$$\begin{aligned}\overline{P}_{j0}^{u} &= E\left\{P_0 r_{j0}^{-\alpha}\right\}\\ &= P_0 E\left\{\left(w_j^2D^2 + r_j^2 - 2w_jDr_j\cos\phi_j\right)^{-(\alpha/2)}\right\} \quad (8)\end{aligned}$$

where the expectation is over a specific region in which MS j may appear. In cases of FCA and traffic adaptive DCA, this region covers a whole cell. The uplink CIR at BS 0 is expressed as

$$(\text{CIR})^{u} = \frac{r^{-\alpha}}{\sum\limits_{j\in\mathcal{I}^u} E\left\{\left(w_j^2D^2 + r_j^2 - 2w_jDr_j\cos\phi_j\right)^{-(\alpha/2)}\right\}}. \quad (9)$$

2) Computation of CIR and P_{out} for IAMP Method:

a) Downlink CRZ: Under the IAMP method, the border of a downlink CRZ zone-h_k can be found by solving the radius $R_{kd}(\theta)$ for various θ's from the following:

$$\frac{R_{kd}^{-\alpha}}{\sum\limits_{J\in\mathcal{I}_k^d}\left[w_j^2D^2 + R_{kd}^2 - 2w_jDr\cos(\phi_j-\theta)\right]^{-(\alpha/2)}} = \gamma. \quad (10)$$

This equation is obtained by: 1) defining the most sparse cochannel user distribution corresponding to h_k; 2) inserting (7) into (4); and 3) setting the inequality in (4) to equality and setting $r = R_{kd}$. The subscript k of $\mathcal{I}_k^d$ emphasizes that $\mathcal{I}_k^d$ changes with different h_k. The concentric CRZ structure

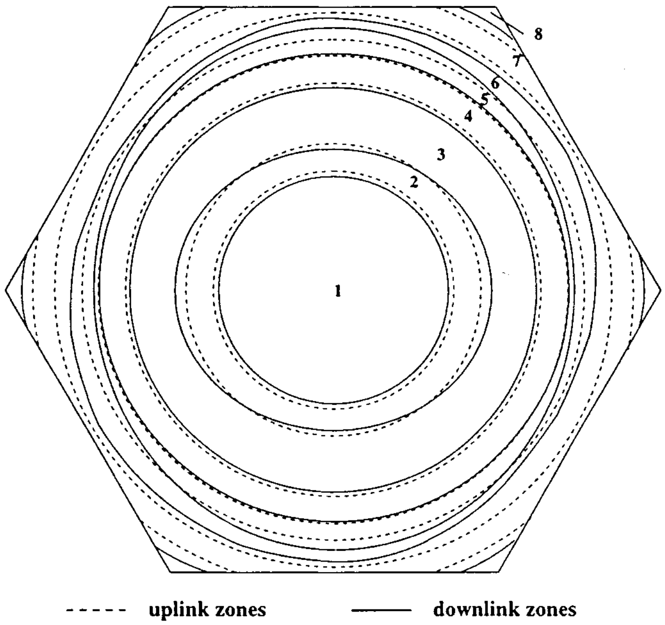

Fig. 5. Concentric CRZ structure with integer h_k's.

is formed due to the fact that interferers move farther as h_k increases.

b) Uplink CRZ: In the case of IAMP, all of the uplink cochannel interference in a zone-h_k channel is from zone-h_k users in other cells. As a result, the expectation in (8) becomes

$$\begin{aligned}\overline{P_j^u} =& P_0 \int_0^{2\pi} \int_{R_{k-1}}^{R_k} \left(w_j^2 D^2 + r_j^2 - 2w_j D r_j \cos\phi_j\right)^{-(\alpha/2)} \\ &\cdot p_k(r_j, \phi_j)\, dr_j\, d\phi_j \\ =& \frac{P_0}{\pi(R_k^2 - R_{k-1}^2)} \int_0^{2pi} \int_{R_{k-1}}^{R_k} \\ &\cdot \left(w_j^2 D^2 + r_j^2 - 2wDr_j \cos\phi_j\right)^{-(\alpha/2)} r_j\, dr_k\, d\phi_j \end{aligned} \tag{11}$$

where $p_k(r_j, \phi_j)$ is the probability density function (pdf) of a zone-h_k user being at (r_j, ϕ_j) (uniform by assumption). Then, the radius of uplink zone-h_k, R_{ku}, can be solved from

$$\begin{aligned}&\frac{\pi\left(R_{ku}^2 - R_{ku-1}^2\right) r^{-\alpha}}{\sum\limits_{j \in \mathcal{I}_k^u} \int_{R_{ku-1}}^{R_{ku}} \left(w_j^2 D^2 + r_j^2 - 2w_j D r_j \cos\phi_j\right)^{-(\alpha/2)} r_j\, dr_j\, d\phi_j} \\ &= \gamma. \end{aligned} \tag{12}$$

Equation (12) is obtained by following a similar procedure as in the downlink case. Note that (12) implies that R_{ku} is independent of θ and thus uplink CRZ's are circles.

Fig. 5 shows the concentric CRZ structure of the IAMP method in the deterministic case, where h_k's are integers and $\alpha = 4.0$ and $\gamma = 18$ dB . By definition, a two-way zone-h_k is the overlap region between downlink zone-h_k and uplink zone-h_k.

The computation of P_{out} is rather straightforward in the CRZ structure. For a specific interference distribution corresponding to h_k, the outage probability P_{out} equals zero within zone-h_k and equals one outside of it.

IV. Cochannel Interference Analysis for Shadowing Environment

In the shadowing environment, the downlink and uplink CIR constraints are expressed as

$$(C/I)^d = \frac{L_0 r^{-\alpha}}{\sum\limits_{J \in \mathcal{I}^d} L_{J0} r_{J0}^{-\alpha}} \geq \gamma \tag{13}$$

and

$$(C/I)^u = \frac{L_0 r^{-\alpha}}{\sum\limits_{j \in \mathcal{I}^u} L_{j0} r_{j0}^{-\alpha}} \geq \gamma \tag{14}$$

respectively. In the deterministic case, the location of a mobile is the sole factor that determines its CIR for the given α and γ parameters. This factor is mapped into CRZ's. In this way, the CIR of a zone-h_k user is guaranteed by assigning a zone-h_k channel to it. The only reason left to block a call is the lack of available channels. However, in the shadowing model P_{out} is no longer only zero or one as in the deterministic model. It is a function of h_k, r, and θ. As a result, the relationship between the geometric borders (and the area) of CRZ's and the number of zone-h_k users is completely destroyed. As we will see later, this relationship plays a pivotal role in the study of performance bound. In order to recover this relationship, the concepts of CRZ and cell have to be revamped as follows. The new definitions call for the terms "the extended cell" and "the extended CRZ" for consistency of terminology, although there no longer exist "zones" in the geographic sense.

A. Extended Cell and Extended CRZ

Regardless of whether there is shadow fading or not, an active MS always has a higher CIR with one of BS's at a given moment. Since the essence of concepts CRZ and cell is to represent the CIR constraint rather than the geometric distance, the definitions should be modified according to (13) and (14).

Definition 3: A **downlink (uplink) extended cell (EC)** *of BS J is a group of active mobiles, with a mean value ξ, which has a higher downlink (uplink) CIR on a channel with BS J than with any other BS.*

Definition 4: A **downlink (uplink) ECRZ** *of h_k, denoted as zone-h_k, is the largest group of mobiles in a downlink (uplink) EC whose* CIR $\geq \gamma$ *on a channel assigned to one out of h_k cells. A* **two-way ECRZ** *of h_k is defined as a group of active mobiles which belong to both the downlink and uplink ECRZ zone- h_k.*

The word "largest" means that if there exist more than one interference distribution patterns for a specific h_k, then zone-h_k corresponds to the one with the lowest P_{out} curve.

Obviously, it is extremely difficult to identify EC's and ECRZ's one by one, even for a snapshot. However, it is not that difficult to find the "area" of an EC or an ECRZ. The "**area**" **of an EC/ECRZ** is defined as the average number of users in the EC/ECRZ at a given time instance. For the purpose of simplicity, let us assume that the solely function of the distance r. Thus, (the downlink) $P_{\text{out}}(r) = P_r\{$the CIR of an MS at distancer $< \gamma\}$. Now the question is: if we put m users at the distance r from a given

BS, what is the average number of successful users who is not blocked? It is straightforward to show that the average number of successful users at distance $r = \sum_{n=0}^{m} n[1 - P_{\text{out}}]$. Therefore, the "area" of ECRZ zone-h_k equals

$$\int_0^{2\pi} \int_0^{H_k} r\left[1 - P_{\text{out}}(h_k,\, r,\, \theta)\right] dr\, d\theta \tag{15}$$

where $P_{\text{out}}(h_k, r, \theta)$ depends upon the specific DCA algorithm. R_k is the upper bound of r, such that $P_{\text{out}}(h_k, R_k) \approx 1$. The "area" of an EC is the summation of "areas" of ECRZ's from one to K.

B. Computation of Cochannel Interference: Shadowing Case

1) CIR Computation for General Case:

a) Downlink CIR: From (6) and (13), the downlink CIR at user 0 in shadowing case is express as

$$(\text{CIR})^d = \frac{L_0 r^{-\alpha}}{\sum_{j \in \mathcal{I}^d} L_{J0}\left[w_j^2 D^2 + r^2 - 2 w_j D r \cos(\phi_j - \theta)\right]^{-(\alpha/2)}}. \tag{16}$$

The major problem in computing P_{out}^d with a shadowing model is dealing with L_{J0}'s. Let

$$e^Y = L_0 r^{-\alpha} \quad e^{Z_J} = L_{J0} r_{J0}^{-\alpha}, \qquad \forall\, J \in \mathcal{I}^d. \tag{17}$$

Since L_j's are assumed to be lognormal RV's, Y and Z_J are both Gaussian with $N(m_y, \sigma^2)$ and $N(m_J, \sigma^2)$, respectively, where $m_y = -\alpha \ln r$ and $m_J = -\alpha \ln r_{J0}$. Moreover, let e^{Z_d} be the summation of lognormal RV's as

$$e^{Z_d} = \sum_{j \in \mathcal{I}^d} e^{Z_J}. \tag{18}$$

Combining (16)–(18), we get

$$P_{\text{out}}^d = P_r\left\{e^Y / e^{Z_d} < \gamma\right\} = P_r\left\{Y - Z_d < \ln\gamma\right\}. \tag{19}$$

Note that e^{Z_d} is a weighted summation of a group of lognormal RV's. There exist three major approaches to approximating such a summation [28], namely, the Wilkinson's approach, Schwartz–Yeh approach, and cumulant matching approach. We use Wilkinson's method in our investigation due to the conclusion of the comparative study in [28] and [29]. In Wilkinson's method, e^{Z_d} is assumed to be another lognormal RV. In other words, Z_d is Gaussian with mean m_{zd} and variance σ_{zd}^2, which are obtained by equating first two moments of (18) as

$$\mu_{1d} = E(e^{Z_d}) = E\left(\sum_{J \in \mathcal{I}^d} e^{Z_J}\right) = e^{\sigma^2/2} \sum_{J=1}^{V_d} r_{J0}^{-\alpha} \tag{20}$$

and

$$\begin{aligned}\mu_{2d} &= E[e^{2Z_d}] \\ &= E\left(\sum_{J \in \mathcal{I}^d} e^{2Z_J}\right) \\ &= e^{2\sigma^2} \sum_{J=1}^{V_d} r_{J0}^{-2\alpha} + 2 e^{\sigma^2} \sum_{J=1}^{V_d} \sum_{S=J+1}^{V_d} r_{J0}^{-\alpha} r_{S0}^{-\alpha}\end{aligned} \tag{21}$$

where V_d is the number of cells in $\mathcal{I}^d$. After μ_{1d} and μ_{2d} are obtained, m_{zd} and σ_{zd}^2 can be found as

$$m_{zd} = 2\ln\mu_{1d} - \tfrac{1}{2}\ln\mu_{2d} \tag{22}$$

$$\sigma_{zd}^2 = \ln\mu_{2d} - 2\ln\mu_{1d}. \tag{23}$$

Since both Y and Z_d are Gaussian, $Y - Z_d$ is also Gaussian with

$$N\left(-m_{zd} - \alpha\ln r,\, \sqrt{\sigma^2 + \sigma_{zd}^2}\right).$$

Then, the downlink outage probability of an MS at (r, θ) is expressed as

$$\begin{aligned}P_{\text{out}}^d(r, \theta) &= P_r\left\{Y - Z_d \le \ln\gamma\right\} \\ &= 1 - Q\left(\frac{\ln\gamma + \alpha\ln r + m_{zd}}{\sqrt{\sigma^2 + \sigma_{zd}^2}}\right).\end{aligned} \tag{24}$$

In (24), r_j can be represented as a function of (r, θ). As a result, m_{zd}, σ_{zd}, and P_{out}^d are functions of (r, θ) if the interference distribution and the parameters (α, γ, σ) are known.

b) Uplink CIR: From (8) and (14), the uplink CIR at BS 0 is

$$(\text{CIR})^u = \frac{L_0 r^{-\alpha}}{\sum_{j \in \mathcal{I}^u} L_{j0} E\left\{\left(w_j^2 D^2 + r_j^2 - 2 w_j D r_j \cos\phi_j\right)^{-(\alpha/2)}\right\}} \tag{25}$$

where the expectation is taken over the region in which MS j may appear. The region depends upon the specific DCA algorithm.

Assuming that

$$\overline{r}_{j0} \triangleq \left[E\left\{\left(w_j^2 D^2 + r_j^2 - 2 w_j D r_j \cos\phi_j\right)^{-(\alpha/2)}\right\}\right]^{-(1/\alpha)} \tag{26}$$

is known for a given DCA algorithm, P_{out}^u can be found in the same way as in the downlink case with obvious changes on indexes

$$P_{\text{out}}^u(r, \overline{r}_{j0}) = 1 - Q\left(\frac{\ln\gamma + \alpha\ln r + m_{zu}}{\sqrt{\sigma^2 + \sigma_{zu}^2}}\right) \tag{27}$$

where

$$m_{zu} = 2\ln\mu_1^u - 0.5\ln\mu_2^u \tag{28}$$

and

$$\sigma_{zu}^2 = \ln\mu_2^u - 2\ln\mu_1^u \tag{29}$$

$$\mu_1^u = e^{\sigma^2/2} \sum_{j=1}^{V_u} \overline{r}_{j0}^{-\alpha} \tag{30}$$

$$\mu_2^u = e^{2\sigma^2} \sum_{j=1}^{V_u} \overline{r}_{j0}^{-2\alpha} + 2 e^{\sigma^2} \sum_{j=1}^{V_u} \sum_{s=j+1}^{V_u} \overline{r}_{j0}^{-\alpha} \overline{r}_{s0}^{-\alpha}. \tag{31}$$

After both P^d_{out} and P^u_{out} are calculated, the two-way outage probability is computed after the definition

$$P_{\text{out}}(r, \theta, \overline{r}_{j0}) = P^d_{\text{out}}(r, \theta) \cup P^u_{\text{out}}(r, \overline{r}_{j0}). \tag{32}$$

2) CIR Computation for IAMP Method: With the IAMP method, the interference on every downlink/uplink zone-h_k channel is from other downlink/uplink zone-h_k channels, and downlink and uplink channels are bound together. Thus, for a zone-h_k user in cell 0, $\mathcal{I}^d_k = \mathcal{I}^u_k \triangleq \mathcal{I}_k$ with interferer number $V_d = V_u \triangleq V_k = V/h_k - 1$. To emphasize the fact that in IAMP method the outage probability depends on h_k, we write P^d_{out} as $P^d_{\text{out}}(h_k, r, \theta)$ and P^u_{out} as $P^u_{\text{out}}(h_k, r, \overline{r}_{j0})$ below.

a) Downlink CIR: With the new notation, P^d_{out} of a zone-h_k user at (r, θ) can be found from (24) with μ_{1d} and μ_{2d} changed to

$$\mu_{1d} = e^{\sigma^2/2} \sum_{J=1}^{V_k} r_{J0}^{-\alpha} \tag{33}$$

and

$$\mu_{2d} = e^{2\sigma^2} \sum_{J=1}^{V_k} r_{J0}^{-2\alpha} + 2e^{\sigma^2} \sum_{J=1}^{V_k} \sum_{S=J+1}^{V_k} r_{J0}^{-\alpha} r_{S0}^{-\alpha} \tag{34}$$

respectively.

b) Uplink CIR: In the uplink case, the expected distance between BS 0 and a zone-h_k user in cell j is calculated from (26) by taking the expectation over ECRZ zone-h_k in cell j (assume uniformly distributed traffic) as shown in (35), given at the bottom of the page. Note that R_k is the upper bound for r_j such that $P^u_{\text{out}}(h_k, r_j) \approx 1$.

Equations (26)–(31) and (35) form a family of equations with unknown function P^u_{out}. R_k in (35) is unknown *a priori*. One way to solve this equation is to search for the solution recursively. A good guess for the initial P^u_{out} is the corresponding P^d_{out} found in (24). The numerical experiment shows that P^u_{out} can be reached within five–six iterations if P^d_{out} is chosen as the initial function.[4]

After both ${}^{d}_{\text{out}}$ and P^u_{out} for IAMP are obtained, the two-way outage probability is computed as the following due to the bound downlink/uplink channels in IAMP

$$\begin{aligned} &P_{\text{out}}(h_k, r, \theta, \overline{r}_{j0}) \\ &\quad = \max\left\{P^d_{\text{out}}(h_k, r, \theta), P^u_{\text{out}}(h_k, r_j, \overline{r}_{j0})\right\}. \end{aligned} \tag{36}$$

[4]The stop condition is the maximum absolute error between two consecutive iterations is less than 10^{-4}.

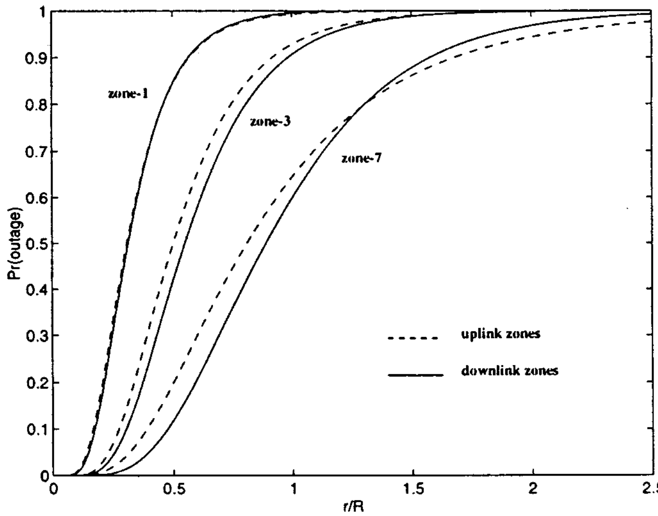

Fig. 6. Probability of outage under shadowing (symmetric interference).

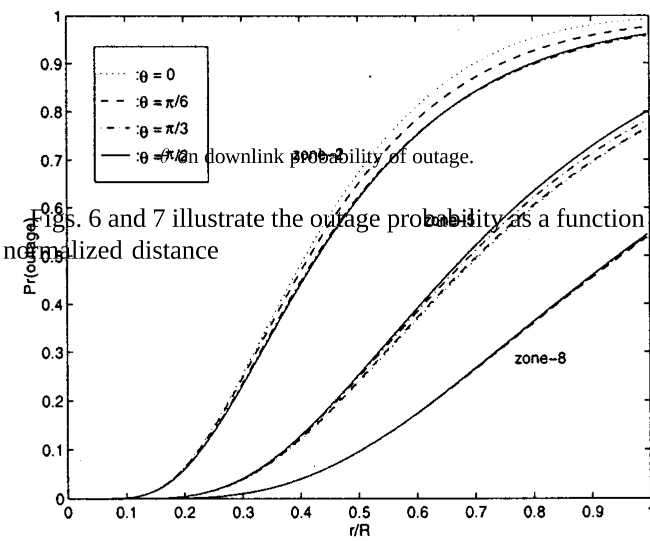

Fig. 7. Effect of θ on downlink probability of outage.

Figs. 6 and 7 illustrate the outage probability as a function of normalized distance r/R for various h_k with the same α, γ , and $\sigma = 8$ dB. Fig. 6 shows both P^d_{out} and P^u_{out} for h_k's whose interference distribution is symmetric and therefore θ has no influence on P_{out}. Note that P^u_{out} is higher than ${}^{d}_{\text{out}}$ in most cases. To appreciate the effect of shadowing, notice that the deterministic case, with $\sigma = 0$, zone-1, zone-3, and zone-7 are circles with normalized radii of 0.357, 0.622, and 0.951, respectively.

$$\begin{aligned} \overline{r}_{j0}(h_k) &= \left\{ \int_0^{2\pi} \int_0^{R_k} \left(w_j^2 D^2 + r_j^2 - 2w_j D r_j \cos\phi_j\right)^{-(\alpha/2)} p_k(r_j, \phi_j)\, dr_j\, d\phi_j \right\}^{-(1/\alpha)} \\ &= \left\{ \frac{\displaystyle\int_0^{2\pi} \int_0^{R_k} \left(w_j^2 D^2 + r_j^2 - 2w_j D r_j \cos\phi_j\right)^{-(\alpha/2)} \left[P^u_{\text{out}}(h_{k-1}, r_j) - P^u_{\text{out}}(h_k, r_j)\right] r_j\, dr_j\, d\phi_j}{\displaystyle 2\pi \int_0^{R_k} \left[P^u_{\text{out}}(h_{k-1}, r_j) - P^u_{\text{out}}(h_k, r_j)\right] r_j\, dr_j} \right\}^{-1/\alpha} \end{aligned} \tag{35}$$

From Fig. 3, it is obvious that because of shadow fading of the signal, a mobile's CIR is no longer guaranteed. For example, a zone-1 user at a distance of $0.3R$ to its BS has an outage probability around 0.5. On the other hand, because of the shadow fading of the interference strength, some users at distances of $0.9 \sim 1.0R$ might have a chance to be packed into zone-1 channels. Such space spreading of users in the same CRZ zone increases as h_k increases. An interesting task here is to check if we gain or lose capacity due to the shadow fading. The change of expected number of users in each zone will be calculated in the next section.

For CRZ's with h_k other than classic channel reuse factors (1, 3, 4, 7, etc.), cochannel interference is not symmetrically distributed. Thus, mobiles with different angles to the BS have different P^d_{out}. The effect of θ on the downlink outage probability for asymmetric interference distributions is shown in Fig. 7. The figure shows that the influence of θ decreases for either less r, because of higher CIR, or larger h_k, due to farther interferer positions.

V. An Asymptotic Perrformance Bound for IA-DCA

As an important application of the CCI computation obtained in previous sections, a lower bound of a performance measure called "the asymptotic probability of assignment failure" is derived for both propagation models. Throughout this section, we assume that: 1) ξ, M are arbitrarily large, but ρ is kept finite and 2) V is large so that the boundary effect is neglected. Since we are considering the asymptotic situation, the content of this section may be called as asymptotic analysis.

As we previously mentioned, the key issue in applying the concept of CRZ/ECRZ in the performance analysis is to establish the relationship between the area of CRZ/ECRZ and the number of users it can contain. Definitions in the first section are used to establish this relationship. In this section, the word zone means either a downlink, an uplink, or a two-way CRZ/ECRZ. P_{out} means either the downlink, uplink, or two-way probability of outage, depending upon what kind of zones are considered. Similarly, theorems proven in this section apply to each of three IA-DCA methods.

A. Definitions

Definition 5: **The asymptotic probability of assignment failure** *is defined as* $\nu^* = \lim_{\xi, M\to\infty} \nu$.

Definition 6: **The area increment of CRZ** *is defined as*

$$A_k = \text{(area of zone-}h_k) - \text{(area of zone-}h_{k-1}) = \int_0^{2\pi}\int_{R_{k-1}(\theta)}^{R_k(\theta)} r\,dr\,d\theta \tag{37}$$

where $R_0 \triangleq 0$. The area change of CRZ is defined as

$$a(h_k) = \lim_{\Delta h_k \to 0} \frac{A_k = A_{k-1}}{\Delta h_k} \tag{38}$$

where $\Delta h_k = h_k - h_k - 1$.

Definition 7: **The extended area increment of ECRZ** *is defined as*

$$A^s_k = \int_0^{2\pi}\int_0^{R_k(\theta)} [P_{\text{out}}(h_{k-1}, r, \theta) - P_{\text{out}}(h_k, r, \theta)]\, r\,dr\,d\theta \tag{39}$$

where $P_{\text{out}}(h_0, r) \triangleq 1$ *and* s *stands for "shadowing."* R_k *is the upper bound of* r*, such that* $P_{\text{out}}(h_k, R_k) \approx 1$. **The extended area change of ECRZ** is expressed as

$$a^3(h_k) = \lim_{\Delta h_k \to 0} \frac{A^s_k - A^s_{k-1}}{\Delta h_k}. \tag{40}$$

B. Lower Bound of ν^* for Deterministic Model

Theorem 3: The lower bound of the asymptotic probability of assignment failure for IA-DCA in a deterministic case can be approximately represented as

$$\nu^*_{\min} = \begin{cases} 0, & \rho < 1/\eta(K) \\ \dfrac{1}{h_q}\left[\eta(q-1) - \dfrac{1}{\rho}\right] + \zeta(q), & 1/\eta(q) < \rho \le 1/\eta(q-1),\ q = 2, \cdots, K \\ 1 - \dfrac{1}{\rho}, & \rho > 1/\eta(1) \end{cases} \tag{41}$$

where

$$\eta(q) = \frac{1}{A_c}\sum_{k=1}^{q} h_k/A_k \quad \zeta(q) = \frac{1}{A_c}\sum_{k=q}^{K} A_k. \tag{42}$$

Proof: Since U_j is uniformly distributed over a cell

$$\text{(Avg. \# of CRZ zone-}h_k \text{ users in system)} = \xi V A_k/A_c. \tag{43}$$

The maximum number of zone-h_k users a channel supports is V/h_k. Hence, the average number of channels needed to support zone-h_k users is $\xi h_k/A_k/A_c$. Thus

$$\text{total \# of channels needed} = \sum_{k=1}^{K} \frac{\xi h_k A_k}{A_c} = \xi\eta(K).$$

Let us define F_k = the number of failed zone-h_k users. Due to the ordering of channel packing in IAMP algorithm, a zone-h_k user cannot fail unless all zone-$h_l (l > k)$ users have already failed. Therefore, when the number of available channels $M \ge \xi\eta(K)$, $\nu^*_{\min} = 0$. And when $\xi\eta(K) > M \ge \xi\eta(K-1)$

$$\begin{aligned} E[F] &= E[F_K] \\ &= \text{(zone-}h_k \text{ channel deficit)} \\ &\quad (\text{\# of zone-}h_k \text{ users per channel}) \\ &= \left(\frac{\xi}{A_c}\sum_{k=1}^{K} h_k A - k - M\right)\frac{V}{h_K} \\ &= \xi V\left\{\left(\frac{1}{A_c}\sum_{k=1}^{K-1} h_k A_k - \frac{M}{\xi}\right)\frac{1}{h_K} + \frac{A_K}{A_c}\right\} \\ &= \xi V\left\{\frac{1}{h_K}\left(\eta(K-1) - \frac{1}{\rho}\right) + \zeta(K)\right\}. \end{aligned}$$

For the case where $\xi\eta(K-1) > M \geq \xi\eta(K-2)$

$$\begin{aligned} E[F] &= E[F_{K-1}] + E[F_K] \\ &= \left(\frac{\xi}{A_c}\sum_{k=1}^{K-1} h_k A - k - M\right)\frac{V}{h_{K-1}} + \xi V \frac{A_K}{A_c} \\ &= \xi V \left\{\left(\frac{1}{A_c}\sum_{k=1}^{K-2} h_k A_k = \frac{M}{\xi}\right)\frac{1}{h_{K-1}} \right. \\ &\qquad \left. + \frac{A_K + A_{k-1}}{a_C}\right\} \\ \gamma &= \xi V \left\{\frac{1}{h_{K-1}}\left(\eta(K-2) - \frac{1}{\rho}\right) + \zeta(K-1)\right\}. \end{aligned}$$

In general, when $\xi\eta(q) > M \geq \xi\eta(q-1)$, $q = 2, 3, \cdots, K$

$$E[F] = \sum_{k=q}^{K} E[F_K] = \xi V \left\{\frac{1}{h_q}\left(\eta(q-1) - \frac{1}{\rho}\right) + \zeta(q)\right\}.$$

Finally, when $M < \xi\eta(1)$

$$\begin{aligned} E[F] &= \sum_{k=1}^{K} E[F_K] \\ &= \xi V \left\{\left(\frac{A_1}{A_c} - \frac{M}{\xi}\right)\frac{1}{1} + \frac{A_2 + \cdots + A_K}{A_c}\right\} \\ &= \xi V \left(1 - \frac{1}{\rho}\right). \end{aligned}$$

Equation (41) is obtained via the definition of ν^*.

Theorem 4: The lower bound of the asymptotic probability of assignment failure of IA-DCA in deterministic case is

$$\nu^*_{\min} = \begin{cases} 0, & \rho < 1/\eta(K) \\ \zeta(q), & 1/\eta(K) < \rho \leq 1/\eta(1) \\ 1 - \dfrac{1}{\rho}, & \rho > 1/\eta(1) \end{cases} \tag{44}$$

where

$$\begin{aligned} \zeta(q) &= \frac{1}{A_c}\int_{h_q}^{h_K} a(x)\,dx \\ \eta(q) &= \frac{1}{A_c}\left[A_1 + \int_1^{h_q} xa(x)\,dx\right] \end{aligned} \tag{45}$$

and h_K and h_q are defined, respectively, by

$$\int_1^{h_K} a(x)\,dx = A_c - A_1 \quad \int_1^{h_q} xa(x)\,dx = \frac{A_c}{\rho} - A_1. \tag{46}$$

Proof: Using the definition of *the area change of* CRZ and noticing that $\eta(k) - \eta(k-1) \to 0$ as $K \to \infty$, one can obtain the above theorem in a straightforward way.

C. Lower Bound of ν^* for Shadowing Model

The major impact of shadow fading on performance analysis is the damage to the simple relationship of (43) in the deterministic case. However, whenever P_{out} is found, the relationship is recovered by the introduction of ECRZ as shown in the following proposition.

Theorem 5: The lower bound for the asymptotic probability of assignment failure of IA-DCA under lognormal shadowing can be approximately represented as

$$\nu^*_{\min} = \begin{cases} 0, & \rho < 1/\eta(K) \\ \dfrac{1}{h_q}\left[\eta(q-1) - \dfrac{1}{\rho}\right] & \\ \quad +\zeta(q), & 1/\eta(q) < \rho \leq 1/\eta(q-1), \\ & \quad q = 2, \cdots, K \\ 1 - \dfrac{1}{\rho}, & \rho > 1/\eta(1) \end{cases} \tag{47}$$

where

$$\eta(q) = \frac{1}{A_c}\sum_{k=1}^{q} h_k A_k^s \quad \zeta(q) = \frac{1}{A_c}\sum_{k=q}^{K} A_k^s. \tag{48}$$

Proof: A tiny region in which $P_{\text{out}}(h_k, r, \theta)$ is approximately constant for a fixed h_k has an area $\Delta A = r\Delta r\Delta\theta$. Since U_j is uniformly distributed over a cell, the average number of active users in ΔA is $\xi\Delta A/A_c$. Therefore, average number of zone-h_k users in the system is expressed as

$$\begin{aligned} &= \lim_{\Delta A \to 0}\sum_{\Delta A}\frac{\xi V}{A_c}\Delta A[1 - P_{\text{out}}(h_k, r, \theta) \\ &\quad - (1 - P_{\text{out}}(h_{k-1}, r, \theta))] \\ &= \frac{\xi V}{A_c}\int_0^{2\pi}\int_0^{R_k}[P_{\text{out}}(h_{k-1}, r, \theta) - P_o(h_k, r, \theta)]r\,dr\,d\theta \\ &= \xi V A_k^s/A_c. \end{aligned} \tag{49}$$

The rest of the proof follows a similar way to the one used to prove Theorem 3 in deterministic case by making obvious changes on A_k and $a(h_k)$.

The final proposition can be obtained by introducing the definition of $a^s(h_k)$ and letting $K \to \infty$, as in the deterministic case.

Theorem 6: The lower bound for the asymptotic probability of assignment failure of IA-DCA for the shadowing model is

$$\nu^*_{\min} = \begin{cases} 0, & \rho < 1/\eta(K) \\ \zeta(q), & 1/\eta(K) \geq \rho < 1/\eta(1) \\ 1 - \dfrac{1}{\rho}, & \rho > 1/\eta(1) \end{cases} \tag{50}$$

where

$$\begin{aligned} \zeta(q) &= \frac{1}{A_c}\int_{h_q}^{h_k} a^s(x)\,dx \\ \eta(q) &= \frac{1}{A_c}\left[A_1^s + \int_1^{h_q} xa^s(x)\,dx\right] \end{aligned} \tag{51}$$

and h_K and h_q are determined, respectively, by

$$\int_1^{h_K} a^s(x)\,dx = A_c - A_1^s \quad \int_1^{h_q} xa^s(x)\,dx = \frac{A_c}{\rho} - A_1^s. \tag{52}$$

VI. Numerical Results and Discussions

Theorems 3 and 5 give concise expressions for the lower bounds of ν^* for the interference adaptive DCA schemes. However, computation of $\nu^*_{\min}$ through these propositions

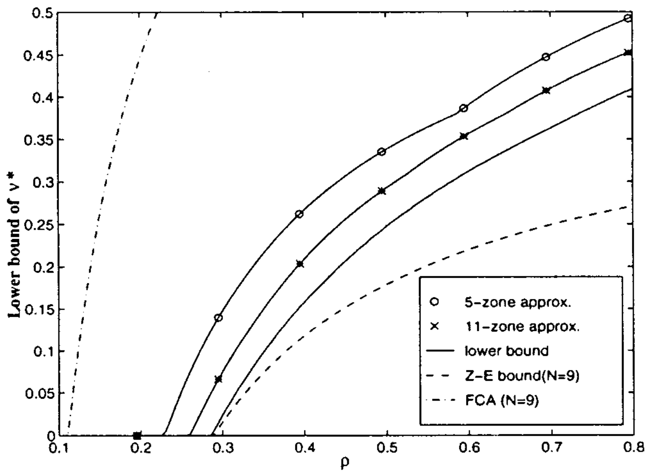

Fig. 8. Comparison of deterministic downlink lower bound with Zander–Ericksson's result.

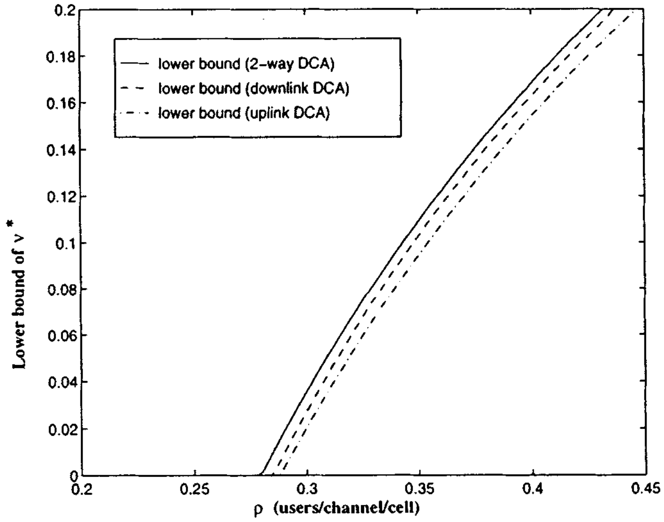

Fig. 9. Lower bound of ν^* for deterministic model.

is nontrivial due to the calculation of $a(h_k)/a^s(h_k)$. A closed-form solution for $a(h_k)$ and $a^s(h_k)$ is still a topic of future study. The results shown in this section are obtained numerically. All of bounds are computed for $\gamma = 18$ dB and $\alpha = 4.0$. In the shadowing model, σ is assumed to be 8 dB.

To compare with the lower bound of reuse partitioning DCA and ν^* of FCA proposed by Zander and Eriksson [20] (with channel reuse factor $N = 9$).[5] Fig. 8 depicts the downlink lower bound for ν^*. Two approximate bounds obtained from Theorem 2 are also shown. The lower bound is calculated from Theorem 3. The $a(h_k)$ is obtained by interpolation. It is fairly understandable that the two bounds are very close when ρ is low, since in [20] the assumption of continuous reuse partitioning is used, which is impractical in location adaptive DCA, but is implemented by interference adaptive DCA. For a higher value of ρ, the Zander–Eriksson's bound is too loose in the sense that ν^* goes up to about 0.65 rather than 1 (as in proposed bound) when ρ goes to infinity. It should be pointed out that ξ is pushed to infinity in the definition of ν^*. As a result, the traffic dynamics among cells are eliminated. On the other hand, it is well known that FCA outperforms all traffic adaptive DCA schemes when ξ is very large. Therefore, it is appropriate to view the difference between the bound of IA-DCA and FCA in Fig. 8 as the gain from a more accurate representation of the CIR constraint in IA-DCA than in TA-DCA.

The lower bounds of ν^* for the deterministic case and the shadowing case are displayed in Figs. 9 and 10, respectively. Comparison of the lower bounds in these figures shows that lognormal shadowing has some positive impact on the system capacity. For example, if we observe two downlink bounds at $\nu^* = 1\%$, the average traffic load ρ (*user/channel/cell*) of the system is approximately 0.36 in Fig. 10 and 0.29 in Fig. 9. This implies that 20% $\sim$ 25% capacity gain can be obtained by utilization of shadowing with a typical value of variance ($\sigma = 8$ dB) for IA-DCA strategies. Even with two-way bounds, there is still about 8% $\sim$ 10% capacity gain. This theoretical conclusion is in agreement with the simulation results reported in

[5] $\gamma = 18$ dB and $\alpha = 4.0$ corresponds to $h_k \approx 7.22$. Only the downlink bound in the deterministic case is available for comparison.

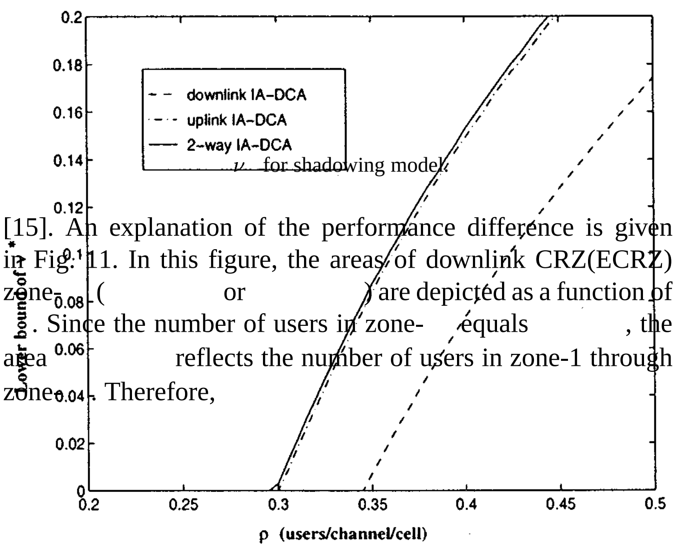

Fig. 10. Lower bound of ν^* for shadowing model.

[15]. An explanation of the performance difference is given in Fig. 11. In this figure, the areas of downlink CRZ(ECRZ) zone- (or) are depicted as a function of . Since the number of users in zone- equals , the area reflects the number of users in zone-1 through zone- . Therefore,

h_k $\sum_{n=1}^{k} A_n$ $\sum_{n=1}^{k} A_n^s$ h_k h_k $\xi A_k/A_c$ $\sum_{n=1}^{k} A_n$ h_k $(\sum_{n=1}^{K} A_n)/A_c = 1$ implies that in order to handle the group of users with average number ξ, one needs up to K CRZ's with the largest reuse factor being h_K. From Fig. 11, it is known that the area of zones in ECRZ is always larger than the one in CRZ at the same h_k. The value of h_K decreases from about 7.22 in the deterministic model to 5.56 in the shadowing case. This increase in A_k (or decrease in h_K) results in the improvement of the bound for ν^*.

Other conclusions that can be drawn from these figures include: 1) in the deterministic case, three bounds are very close to each other. This implies that interference on downlink and uplink channels are similar if channels are allocated by IAMP method and 2) in the shadowing case, the uplink

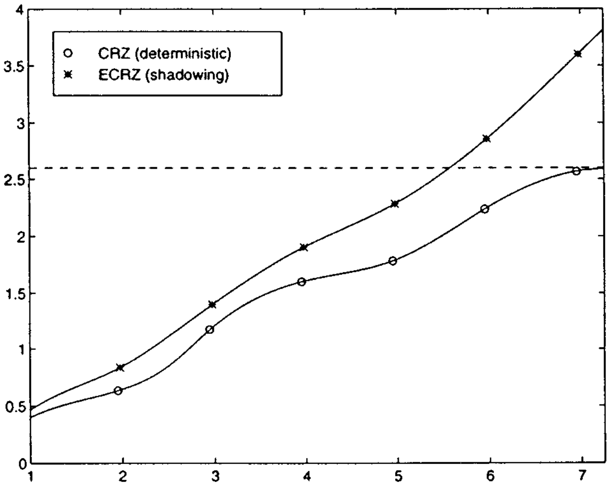

Fig. 11. Area change in CRZ and ECRZ.

cochannel interference is the main limit to the capacity of the TDMA/FDMA system using IA-DCA schemes.

VII. Conclusions

In this paper, the computation of cochannel interference and outage probabilities is presented for both downlink and uplink channels in the TDMA/FDMA system using interference adaptive DCA schemes. This computation is critical in the analysis of IA-DCA schemes. As an important application, the lower bound of the asymptotic probability of assignment failure is calculated for the downlink/uplink-only as well as two-way balanced interference adaptive DCA strategies. Both the deterministic and the shadowing propagation models are considered. The bound is shown to be a closed-form function of the traffic load ρ. The effects of CIR threshold γ and the path-loss exponent α can only be observed in the numerical way. It is analytically shown that a capacity enhancement is possible by utilizing the shadow fading in DCA schemes. This capacity improvement cannot be achieved by either traffic adaptive or location adaptive DCA schemes, while it can be realized by interference adaptive DCA schemes.

Our research plan is to find the performance bound of a real TDMA/FDMA system with the IA-DCA strategy by alleviating the assumption of arbitrarily large M. Ongoing work also includes the application of the results in this paper to be used for more realistic traffic models, such as nonuniform traffic distribution.

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